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FLOW PATTERN MAPS FOR SOLID-LIQUID FLOW IN PIPES

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Abstract—Various flow patterns can be encountered in a solid-liquid pipeline, which affect the hydrodynamic characteristics of the flow. A method for displaying their range of existence by means of flow maps is presented. The maps are drawn from the results of a phenomenological model. The effect of the various operational conditions on the flow pattern transitions are obtained easily using the maps.

Key Words: solid-liquid, flow patterns

INTRODUCTION

Solid-liquid mixtures are encountered in various industries. The interest in this mode of transportation has grown in recent years since it may prove to be an attractive alternative to other modes of transport. It has several advantages, such as its friendliness to the environment, the relatively little infrastructure work needed and possible low operation and maintenance costs.

In most cases the transported slurries consist of small size particles (usually up to a few tens of microns). However, coal-water mixtures and sand or gravel slurries could contain much coarser particles (up to a few millimeters). These mixtures, and also mixtures which contain smaller particles having high density (such as iron), are considered as settling slurries. They cannot be treated as pseudo-fluids with effective rheological properties, since the solid particles tend to accumulate at the bottom of the pipe, which gives rise to several flow patterns. The flow patterns, which may be encountered in a solid-liquid pipeline, affect the dependence of the pressure drop on the flow rate as well as the magnitude of the pressure drop and they also influence pipe erosion and other performance characteristics.

Since the flow is very complex and definition of the flow patterns relies mostly on visual observations, various reports assign different names to certain characteristics, especially the transition velocities between the flow patterns.

One of the most common classifications (Vocaldo & Charles 1972; Parzonka *et al.* 1981) relates to four flow patterns—"homogeneous flow", "heterogeneous flow", "heterogeneous and sliding-bed flow" and "saltation and stationary bed flow" (Ayazi Shamlou 1987). It is important to note, that saltation can also be associated with the moving bed flow pattern. Bain & Bonnington (1970) and Turian & Yuan (1977) even used the term "saltation" for the flow pattern referred to as a "moving bed" in this work. Some investigators used cruder distinctions. For example, Durand (1953) and Condolios & Chapus (1963) referred to a "non-deposit flow regime" and a "regime with deposits". Brown (1991) differentiated between a "fully-segregated flow regime", where all the particles are present in the bed, and a "heterogeneous flow regime", which actually includes flow with a bed and with a heterogeneous suspension. In other cases the division is finer. Lazarus & Neilson (1978) listed a "stationary bed", "part stationary bed", "fully moving bed", "heterogeneous flow", "pseudohomogeneous flow" and "homogeneous flow", the last three being parts of the "fully suspended flow". In the experimental work of Ercolani *et al.* (1979) they tried to delineate "pseudohomogeneous flow", "heterogeneous flow", "limit deposit condition", "moving-stationary bed", "moving dunes" and "stationary bed".

The transitions between the flow patterns are usually determined by visual observations (whose inherently limited reliability is enhanced by the gradual nature of the transitions) and naturally their definition depends on the definition of the flow patterns. Thus, the terminology for the transition velocities is quite confusing.

Most of the confusion is associated with the transition from the stationary bed flow pattern, since some investigators claim that it is associated with the velocity at minimal pressure gradient. In most cases this is not true, as observed as early as 1970 by Wasp *et al.*, who claimed: "The minimum in the pressure loss vs velocity curve has nothing to do with the transition velocities."

The term "limit deposit velocity" has been used in quite a few works, e.g. Wilson (1972 and since) and Toda *et al.* (1980), for the stationary bed limit. Durand (1953) used it to mark the separation between the deposit and non-deposit regimes. Other investigators called this transition velocity the "critical deposit velocity" (Graf *et al.* 1970; Bain & Bonnington 1970; Stevens & Charles 1972; Kazanskij 1979). The term "deposit velocity" was employed for the same purpose by Wood (1979) and Parzonka *et al.* (1981). Shook & Roco (1991) used "deposition velocity". In some cases the "critical velocity" was defined as the velocity below which there are deposited particles (Zandi & Govatos 1967; Vocaldo & Charles 1972; Goedde 1978; Ercolani *et al.* 1979; Oroskar & Turian 1980; Turian *et al.* 1987). However, the same term was associated with the minimal pressure drop by

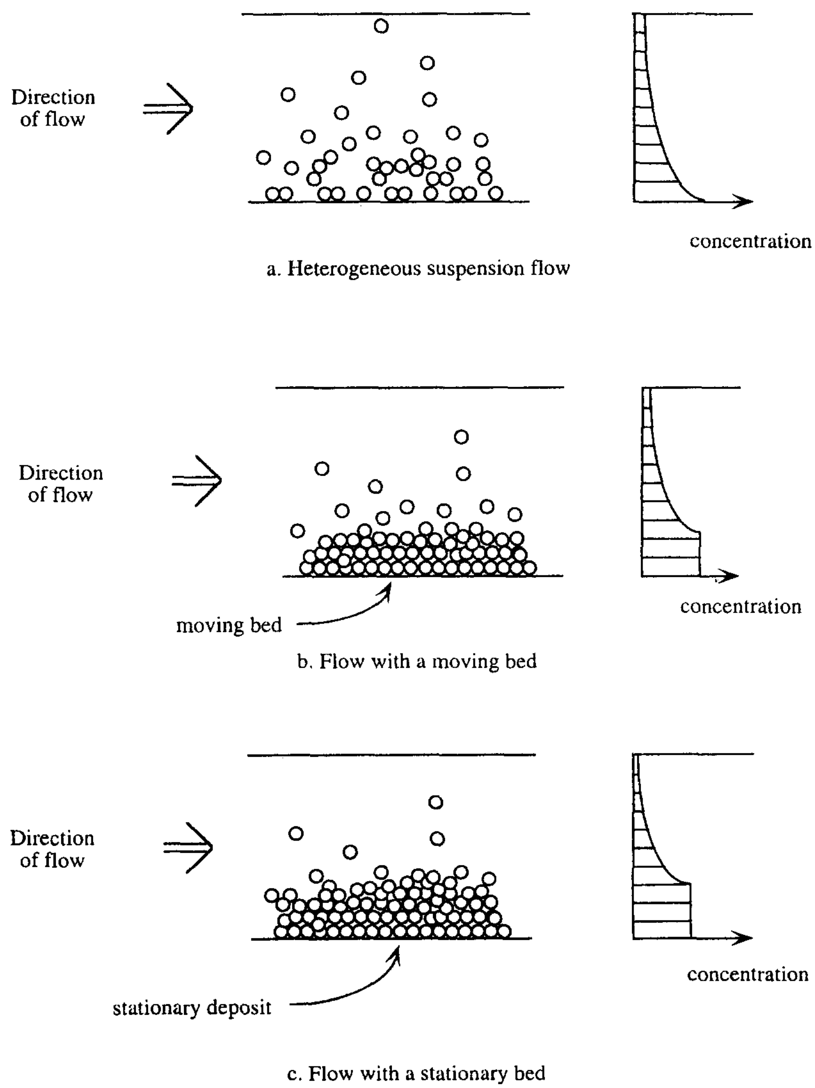

Figure 1. Schematic views of flow patterns and concentration distributions in a direction perpendicular to the pipe axis.

Smith (1955), Bain & Bonnington (1970) and Takaoka *et al.* (1980). Other terms which were used for the transition to fully suspended flow were "settling velocity" (Smith 1955) and "minimum velocity" (Spells 1955).

In the present work we have attempted to group together flow patterns which have similar behavior and to refer to their most conspicuous characteristics regarding the distribution of the solids in the pipe. Thus, three main flow patterns are defined:

(1) *Fully suspended flow*—at high mixture flow rates all the solid particles are suspended. The fully suspended flow pattern may be subdivided into two sub-patterns: (a) pseudohomogeneous suspension, when the solids are distributed nearly uniformly across the pipe cross-section. The mixture velocities required for such flow are usually very high and cannot be considered practical; (b) heterogeneous suspension flow, when there is a concentration gradient in the direction perpendicular to the pipe axis, with more particles transported at the lower part of the pipe cross-section [figure 1(a)]. This is the case in most practical applications.

(2) *Flow with a moving bed*—at lower mixture flow rates solid particles accumulate at the bottom of the pipe [figure 1(b)]. Thus they form a packed bed layer, which moves along the pipe bottom. The concentration of this layer corresponds to maximal packing, or nearly so. The upper part of the pipe cross-section is occupied by a heterogeneous mixture.

(3) *Flow with a stationary bed*—when the mixture flow rate is too low to enable motion of all immersed particles, a stationary deposit is observed at the bottom of the pipe [figure 1(c)]. On top of this deposit particles are transported as a separate moving layer. In many cases one observes dune-like forms on the upper part of the bed, a phenomenon known as "saltation". The rest of the pipe is still occupied by a heterogeneous mixture, though its concentration profile is much steeper than in the other flow patterns.

The following terminology is employed in this work for the significant velocities:

(1) *Limit deposit velocity*—the velocity that demarcates the upper limit of stationary bed existence.

(2) *Suspending velocity*—the velocity above which the flow is fully suspended.

The prediction of the flow pattern which would exist in the pipe for any given set of operational conditions is very important. One would like to avoid the formation of a stationary deposit, which causes partial blockage of the pipe, thus reducing its efficiency. A bed layer probably enhances pipe wear. It may also cause non-uniform heat transfer characteristics around the pipe periphery, which should usually be avoided. Moreover, the pressure drop behavior is different from one flow pattern to another (as mentioned before). In this paper the three-layer model of Doron & Barnea (1993) is employed for the drawing of flow pattern maps, which can be used to indicate the flow pattern and the effect of the operational variables on the transition lines.

THE THREE-LAYER MODEL

The three-layer model for the prediction of hydrodynamic characteristics for pipe flow of solid–liquid mixtures containing relatively coarse particles was presented in detail by Doron & Barnea (1993), and is reviewed briefly in this section. All the constitutive relations and geometrical expressions used in the equations listed below can be found in Doron & Barnea (1993).

At low mixture velocities the flow is considered as consisting of three layers—a stationary deposit at the bottom of the pipe, a moving bed layer above it and an upper layer which contains a heterogeneous suspended mixture. The height of the stationary bed is postulated to be such that the velocity of the moving bed above it is at a certain minimal value. When the moving bed velocity assumes this value, the particles at the interface between the two bed layers are at the verge of rolling. For this condition the driving torque acting on a particle at the lowermost stratum of the moving layer (i.e. right at the interface between the stationary layer and the moving layer) should be equal to the opposing torque acting on that particle. The torque balance yields the required minimal bed velocity, U_{bc}:

$$U_{bc} = \sqrt{\frac{1.56(\rho_S - \rho_L)gd_p\left[\sin\left(\frac{\pi}{6}\right) + \frac{1}{2}C_{mb}\left(\frac{y_{mb}}{d_p} - 1\right)\right]}{\rho_L C_D}} \qquad [1]$$

where ρ_S and ρ_L are the densities of the solids and of the liquid, respectively, g is the gravitational acceleration, d_p is the diameter of the solid particles, C_{mb} is the concentration of the solids in the moving bed, y_{mb} is the height of the moving bed and C_D is the particle drag coefficient (based on U_{bc}).

Consider a case where the operational conditions would correspond to two-layer flow with a moving bed mean velocity which is smaller than the minimum, U_{bc}. In this case, a stationary layer is formed at the bottom of the pipe, and the velocity of the moving bed layer, U_{mb}, is equal to the minimal bed velocity, U_{bc}.

Two continuity equations are written for the two phases:

$$U_h C_h A_h + U_{mb} C_{mb} A_{mb} = U_s C_s A \qquad [2]$$

for the solid particles, and

$$U_h(1 - C_h)A_h + U_{mb}(1 - C_{mb})A_{mb} = U_s(1 - C_s)A \qquad [3]$$

for the liquid phase, where U is the mean velocity, C is the solids volumetric concentration, A is the cross-sectional area and the subscripts h, mb and s denote the upper heterogeneous layer, the moving bed layer and the delivered mixture, respectively.

Force balances (for the components parallel to the mean flow velocity) are written for each layer. The heterogeneous mixture in the upper dispersed layer is considered as a pseudo-liquid with effective properties. Hence:

$$A_h \frac{dP}{dx} = -\tau_h S_h - \tau_{hmb} S_{hmb} \qquad [4]$$

where dP/dx is the pressure gradient, τ_h is the hydraulic shear stress acting on the perimeter of the heterogeneous layer, S_h, and τ_{hmb} is the shear stress acting on the interface between the heterogeneous layer and the moving bed layer, S_{hmb}.

For the moving bed layer the force balance yields:

$$A_{mb} \frac{dP}{dx} = -F_{mbsb} - \tau_{mbsb} S_{mbsb} - F_{mb} - \tau_{mb} S_{mb} + \tau_{hmb} S_{hmb} \qquad [5]$$

where F_{mbsb} is the dry friction force which acts at the interface between the moving bed layer and the stationary bed layer, S_{mbsb}, and τ_{mbsb} is the hydraulic shear stress acting on that interface, F_{mb} is the dry friction force which acts at the perimeter of the moving bed layer, S_{mb}, and τ_{mb} is the hydraulic shear stress acting on that perimeter.

The force balance on the stationary bed is not part of the solution process, as it only yields an inequality, which serves as verification for the existence of this layer:

$$A_{sb} \frac{dP}{dx} + F_{mbsb} + \tau_{mbsb} S_{mbsb} \leqslant F_{sb} \qquad [6]$$

where A_{sb} is the cross-sectional area of the stationary bed and F_{sb} is the dry friction force which acts at its perimeter.

The mechanism which governs the dispersion of the solid particles in the upper layer is represented by the well-known diffusion equation. Integration over the upper layer cross-section yields the mean concentration in that layer:

$$\frac{C_h}{C_{mb}} = \frac{D^2}{2A_h} \int_{\theta_{sb} + \theta_{mb}}^{\pi/2} e^{-\frac{wD}{2\epsilon}[\sin\gamma - \sin(\theta_{sb} + \theta_{mb})]} \cos^2\gamma \, d\gamma \qquad [7]$$

where D is the diameter of the pipe, w is the terminal settling velocity of the solid particles, ϵ is the diffusion coefficient and θ_{sb} and θ_{mb} are the central angles associated with the stationary bed and with the moving bed, respectively.

All the terms in the six-equation set ([1]–[5], [7]) can be expressed in terms of the mean velocity of the upper layer, U_h, the mean velocity of the moving bed, U_{mb}, the mean concentration of the upper layer, C_h, the height of the moving bed, y_{mb}, the height of the stationary bed, y_{sb}, and the pressure gradient, dP/dx (see Doron & Barnea 1993). These state variables can then be obtained for any given set of operational conditions.

The transitions between the flow patterns are found from the model as follows: at a low mixture flow rate the height of the stationary bed layer, y_{sb}, and the moving bed layer, y_{mb}, are obtained as part of the solution of the model equations. As the flow rate is increased, the calculated stationary bed height diminishes. Transition to flow with a moving bed is associated with the stationary bed height approaching zero. It is reasonable to set the limit deposit condition to the mixture velocity for which $y_{sb} = d_p$. This can be regarded as an upper limit for the limit deposit velocity, U_{LD}, since observations in the laboratory indicate that the stationary bed actually starts moving when its height is several particle diameters. As the flow rate is increased further, the moving bed height is reduced until it approaches one particle diameter ($y_{mb} = d_p$), and transition to fully suspended flow is predicted. It is important to note, that the observed transitions between the flow patterns are not sharp, but rather they occur over a range of flow rates. Thus, the model results can better be regarded as indicative of the transitions and not as absolute values.

The three-layer model results were compared with experimental data, showing satisfactory agreement (Doron & Barnea 1995a, b).

EXPERIMENTAL

The experimental system described in detail by Doron & Barnea (1995b) was used to obtain data on the various characteristics of solid–liquid mixture flow. The main part of the system is a transparent test pipe, which enables visual observations of the flow patterns. The solid particles are 3 mm in diameter acetal spheres, of density 1240 kg/m^3. Most of the particles are black, except for a small amount of white particles, which serve as tracers. The carrier liquid is water.

The mixture volumetric flow rate is measured using an electromagnetic flow meter, and a Coriolis mass flow meter is used to measure the delivered concentration.

The flow pattern is determined by careful observation of the motion of the white tracer particles. When the particles at the lower part of the pipe cross-section do not move, flow with a stationary bed exists. As the mixture flow rate is increased, the stationary bed height is reduced. The limit deposit condition is reached when the particles at the vicinity of the bottom of the pipe start moving

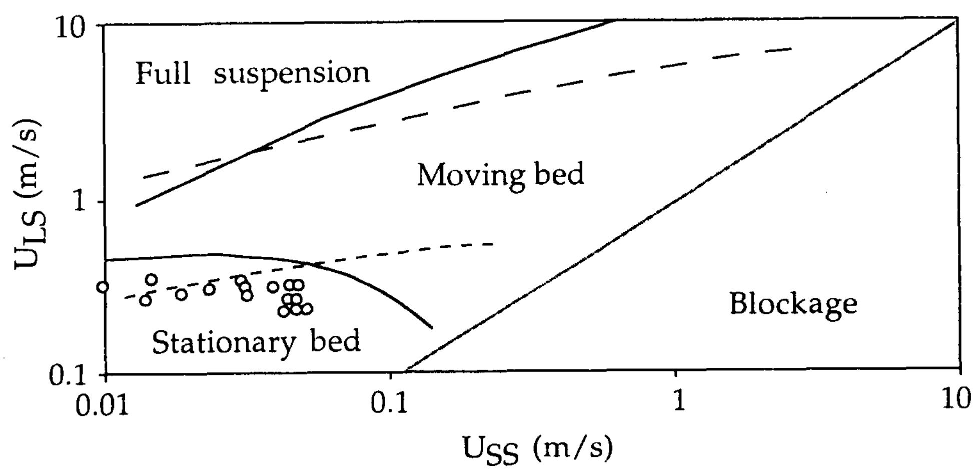

Figure 2. Superficial velocities flow pattern map; $\rho_S = 1240$ kg/m^3, $D = 50$ mm, $d_p = 3$ mm; —— three-layer model, - - -, Turian *et al.* (1987) correlation, — — Turian & Yuan (1977) correlation, ○ experimental data (Doron & Barnea 1995b).

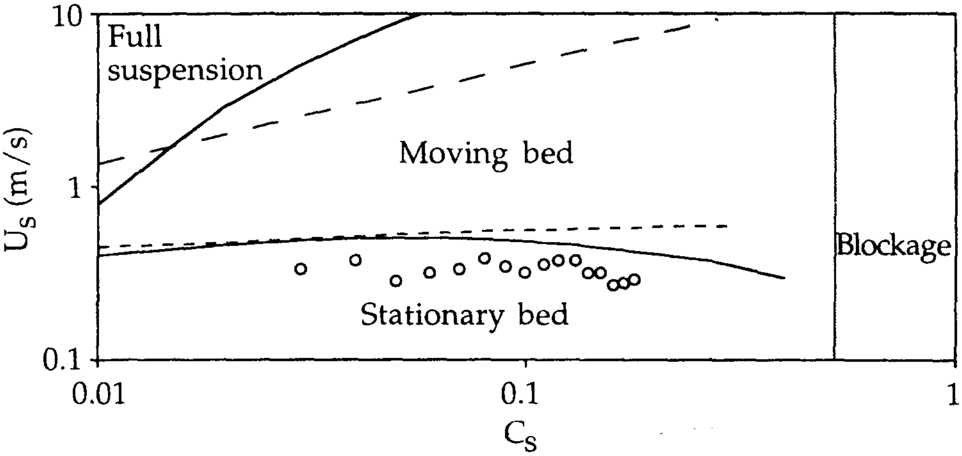

Figure 3. Mixture velocity-delivered concentration flow pattern map, $\rho_S = 1240\,kg/m^3$, $D = 50\,mm$, $d_p = 3\,mm$. For legend, see figure 2.

downstream. This situation is detected easily by observing the white tracers. Increasing the flow rate further leads to a reduction of the moving bed height until the suspending velocity is obtained. For most delivered concentrations this condition could not be attained in our experimental setup because of limitations imposed by the pump capacity and by the structural strength of the rig. Thus, only data regarding the limit deposit velocity are reported.

FLOW PATTERN MAPS

The number of parameters which influence the characteristics of the flow of solid–liquid mixtures is very large. Even for the hydraulic transport of uniform spherical solid particles in a pipe with a circular cross-section they include operational conditions such as pipe diameter and inclination, mixture flow rate and delivered concentration, physical properties of the two phases, i.e. solids size and density, liquid density and viscosity, and other parameters such as dry friction coefficient and angle of internal friction. The presentation of the effect of the various parameters by means of non-dimensional groups is not practical, hence the flow pattern maps are presented with dimensional coordinates. However, it is important to note that since the maps are drawn from the results of a theoretical model, they can be obtained for any set of operational conditions. The maps

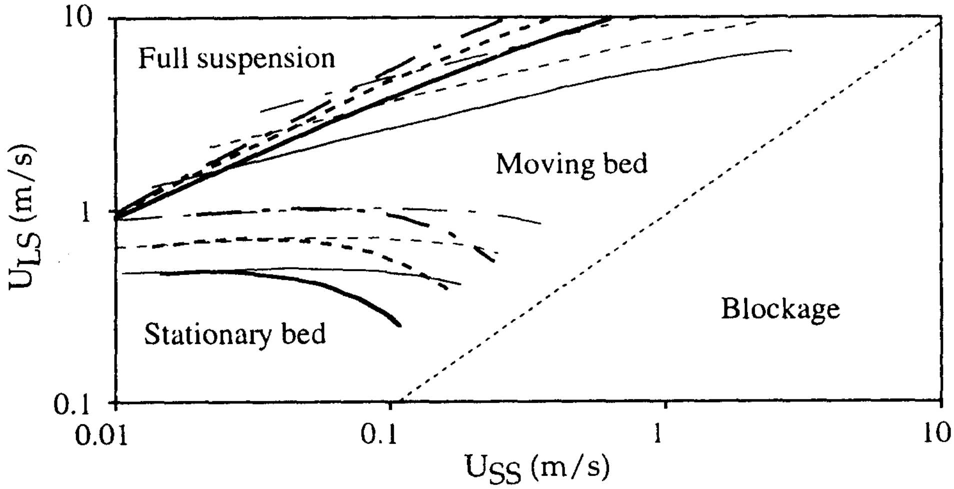

Figure 4. Effect of solids density on superficial velocities flow pattern map; $D = 50\,mm$, $d_p = 3\,mm$; —— $\rho_S = 1240\,kg/m^3$, - - - $\rho_S = 1500\,kg/m^3$, — - — $\rho_S = 2000\,kg/m^3$; thick lines—three-layer model; thin lines—Turian *et al.* (1987), Turian & Yuan (1977) correlations.

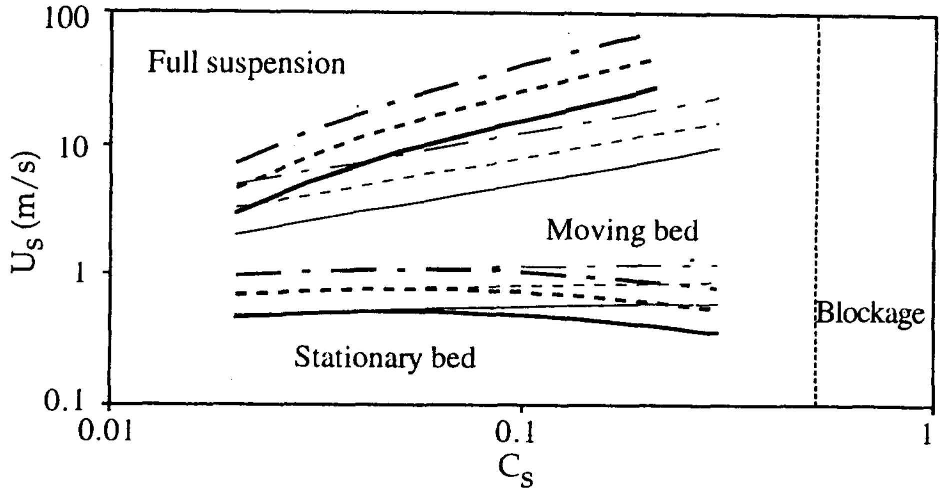

Figure 5. Effect of solids density on the mixture velocity-delivered concentration flow pattern map, $D = 50$ mm, $d_p = 3$ mm. For legend, see figure 4.

shown in this section represent the capability of the method to produce tools which could be useful for pipeline design and operation.

Several modes of presentation of the flow pattern maps are conceivable. Common practice in two-phase gas–liquid flows is to draw the maps in terms of the superficial velocities of the two phases, which are equivalent to the volumetric flow rates. This is also adopted here for solid–liquid flow. Thus, the maps are first presented in terms of the superficial velocities of the solids and the liquid, U_{SS} and U_{LS} respectively (namely, the volumetric flow rate of each phase divided by the pipe cross-section). In many cases the solids and the liquid are fed into the pipe as a mixture. Hence, another type of flow pattern map is presented, namely, in terms of mixture superficial velocity, U_s (i.e. volumetric flow rate of the mixture divided by the pipe cross-section) vs delivered concentration, C_s. The coordinates on the two types of maps are related [$U_{SS} = C_s U_s$, $U_{LS} = (1 - C_s)U_s$ and $U_s = U_{SS} + U_{LS}$, $C_s = U_{SS}/U_{SS} + U_{LS}$]. When the effect of other operational conditions is considered, one may also draw other flow pattern maps. These may be, for example, in terms of mixture flow rate vs tested parameter (for a given delivered concentration) or in terms of mass flow rates of the two phases. All these are actually different presentations of the same results, thus they are not drawn here.

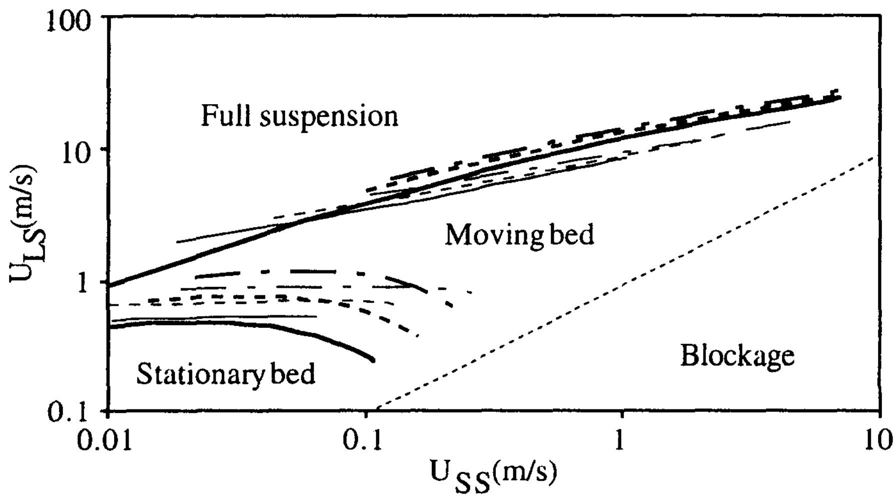

Figure 6. Effect of pipe diameter on superficial velocities flow pattern map, $\rho_S = 1240$ kg/m^3, $d_p = 3$ mm; —— $D = 50$ mm, - - - $D = 100$ mm, — - — $D = 200$ mm; thick lines—three-layer model; thin lines—Turian *et al.* (1987), Turian & Yuan (1977) correlations.

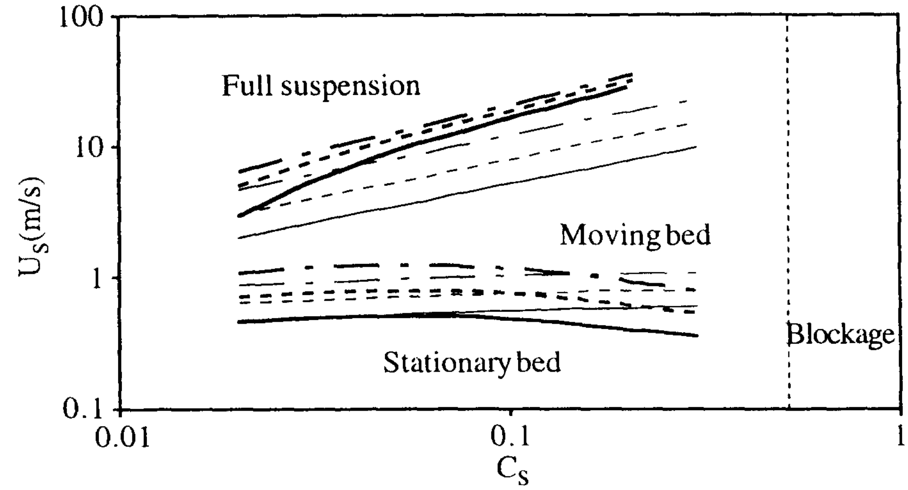

Figure 7. Effect of pipe diameter on the mixture velocity-delivered concentration flow pattern map, $\rho_S = 1240\ \mathrm{kg/m^3}$, $d_p = 3$ mm. For legend, see figure 6.

The transitions between the flow patterns are found using the three-layer model, as described previously. The boundary between flow with a stationary bed and flow with a moving bed is the condition of limit deposit. The transition line from flow with a moving bed to fully suspended flow represents the condition when the height of the moving bed is of the order of one particle diameter, as no lower bed can exist. It is important to note, that the lines presented in the maps are only relatively rough indications of the boundaries between the flow patterns, since in reality the transitions are not sharp and may occur over a range of flow rates. This observation holds especially for the suspending velocity.

A flow pattern map for $\rho_S = 1240\ \mathrm{kg/m^3}$, $D = 50$ mm, $d_p = 3$ mm in terms of the superficial velocities of the solids and of the liquid is presented in figure 2. The boundary between flow with a stationary bed and flow with a moving bed exhibits satisfactory agreement with the new experimental data obtained in our laboratory. Although the data are somewhat overpredicted, the model prediction of the shape of the boundary is better than the Turian *et al.*'s (1987) correlation results, which are also shown in the figure. The theoretical transition from flow with a moving bed to fully suspended flow is presented in the map together with the Turian & Yuan (1977) correlation. As would be expected, the range of existence of the moving bed flow pattern increases with the solids flow rate. When the delivered concentration is set to $C_s = 0.52$, the whole

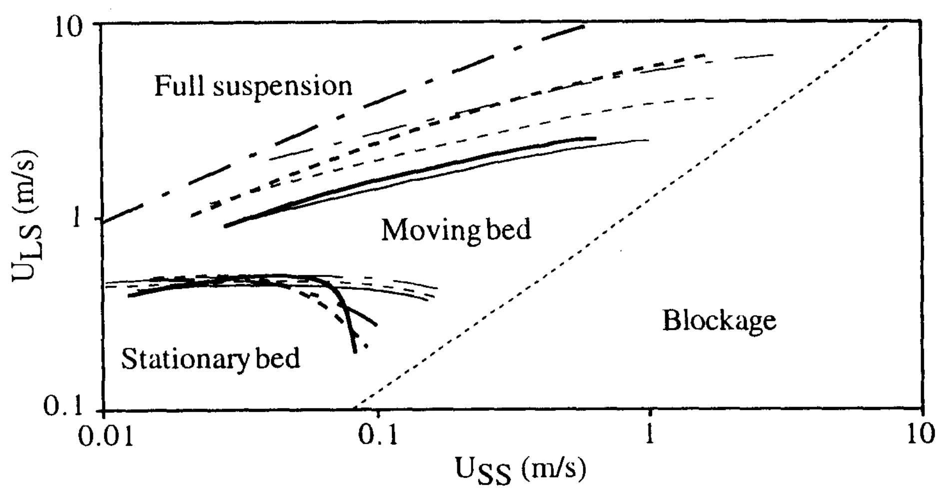

Figure 8. Effect of particle diameter on superficial velocities flow pattern map; $\rho_S = 1240\ \mathrm{kg/m^3}$, $D = 50$ mm; —— $d_p = 0.5$ mm, --- $d_p = 1$ mm, —-— $d_p = 3$ mm; thick lines—three-layer model; thin lines—Turian *et al.* (1987), Turian & Yuan (1977) correlations.

pipe is occupied by a packed mixture. This situation is termed "blockage" here (meaning that the bed fills the whole pipe cross section, although it may still be mobile). The calculated transition lines do not extend all the way to the "blockage" line, since at very high concentrations, the flow is probably governed by a different mechanism than the one represented by the three-layer model.

Figure 3 presents the flow pattern map in terms of mixture superficial velocity vs delivered concentration. In this presentation it is easy to realize that the transition from flow with a stationary bed is relatively independent of the delivered concentration. On the other hand, the region of flow with a moving bed is larger for higher concentrations, since the additional amount of solids in the mixture requires larger flow rates to enable full suspension of the particles.

The effect of some other operational conditions on the flow pattern maps is considered next. In each case, the flow pattern transitions for several values of the examined parameter are plotted on the same chart and the trends are discussed. Each figure relates to variation of a single parameter. The maps are dimensional, hence comparison can be made with data sets for which only this single parameter is varied. Such sets of data are scarce, hence, in order to rely on data banks which are as large as possible, the model results are compared to empirical correlations. The Turian *et al.* (1987) correlation is used to represent the data on the limit deposit velocity and the Turian & Yuan (1977) correlation is used for the data on the suspending velocity. Since the definitions of the flow patterns are ambiguous (as described in the introduction) and due to the inherent limitations of empirical correlations, they should serve to verify trends rather than for quantitative comparisons. Note, that discrepancies between the model results and the correlations are more evident regarding the suspending velocity, where detection of the transition is more difficult.

Figures 4 and 5 show the effect of changing the solids density on the flow pattern boundaries for $D = 50$ mm, $d_p = 3$ mm. The transitions occur at higher mixture flow rates when the solids density is higher. This is reasonable, since the forces required to move the particles and to suspend them are proportional to their weight.

The effect of the pipe diameter on the flow pattern maps is demonstrated in figures 6 and 7 for $\rho_S = 1240$ kg/m^3, $d_p = 3$ mm. As the pipe grows larger, so do the transition velocities. In order to induce the bed motion for stationary–moving bed transition, the mean velocity in the moving bed layer needs to be high enough, a condition which requires a higher flow rate if the pipe cross section is larger. The effect is almost independent of the mixture concentration, hence the limit deposit lines in figure 7 are nearly parallel. Moreover, the dependence of the limit deposit velocity on pipe diameter is almost linear. The results of the model indicate that the normalized bed height (y_{mb}/D) at limit deposit conditions depends very weakly on the pipe diameter. Thus, the minimal bed velocity is almost constant, and the mixture velocity should be proportional only to the pipe diameter. The transition to the fully suspended flow does depend on the delivered concentration (and the curves are not parallel), since it affects the solids content in the upper heterogeneous layer.

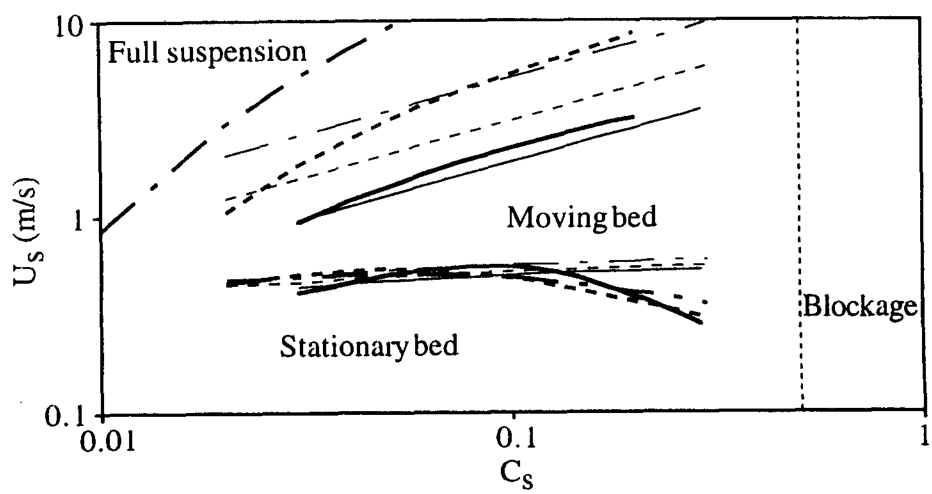

Figure 9. Effect of particle diameter on the mixture velocity-delivered concentration flow pattern map, $\rho_S = 1240$ kg/m^3, $D = 50$ mm. For legend, see figure 8.

The dependence of the flow pattern boundaries on the particle size is presented in figures 8 and 9 for $\rho_S = 1240\ kg/m^3$, $D = 50$ mm. The limit deposit condition is relatively independent of particle diameter (the curves nearly coincide, especially for the larger particles). The transition to fully suspended flow, on the other hand, is affected considerably by the particle size, and the range of existence of the moving bed flow pattern increases for larger particles. This effect results from the higher settling velocities of the larger particles, whose suspension requires higher dispersion coefficients. These can be produced by higher mixture flow rates, and the transition lines are shifted accordingly.

SUMMARY

Flow pattern maps are widely used for gas–liquid flows as a tool for prediction of the flow pattern which would be encountered in a pipe for given operational conditions. For the flow of solid–liquid mixtures in pipes the use of such maps is not common practice. In this paper the potential of flow pattern maps as a tool for research and design of slurry pipelines has been demonstrated.

At the first stage it is required to set clear-cut definitions and terminology for the flow patterns as well as the transitions between them, since one may find a variety of definitions in the literature. The transitions between the flow patterns are then determined by means of the mechanistic three-layer model of Doron & Barnea (1993), and compared to experimental data.

The flow pattern maps were presented in terms of mixture flow rate vs delivered concentration and in terms of flow rates of the two phases (as is common for gas–liquid flows). Some examples for the effect of various operational conditions on the transition lines were presented in order to demonstrate the ability to draw conclusions from examination of the flow patterns maps. Many other presentations are possible, which may be more appropriate for any specific application.

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